

Global Warming: Prices versus Quantities from a Strategic Point of View

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January 16, 2011

Abstract

This paper investigates how the choices of the instruments - quantities or prices - affects the interactions in a stock externality game (global warming) between fossil fuel suppliers and consumers, both cartelized, e.g. OPEC versus IEA (they have tried both instruments in the past and play currently in quantities). More precisely, the paper studies the equilibria in Markov strategies in a dynamic game with each player choosing either the quantity or the price strategy including short-run first mover advantages. The conclusion is that both players should prefer the price instrument. Nevertheless, governments seem to prefer carbon permits to taxes and even OPEC plays in quantities, presumably contrary to its interest. Therefore, both players are expected to switch back to price and tax policies if global warming will be treated effectively.

Keywords: prices versus quantity, carbon taxes versus permits, differential games, Nash and Stackelberg, global warming.

JEL#: D62, Q54, C73.

1 Introduction

This paper investigates the strategic choices of quantity or price instruments in the dynamic context of global warming. It is thus complementary to the

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[†]The author acknowledges helpful discussions with Gerhard Sorger.

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existing literature following the seminal paper of Weitzman (1974) that uses uncertainty to draw a wedge between these instruments that are equivalent in a deterministic and static setting. Hoel and Karp (2001) extend Weitzman's question into a dynamic setting (global warming) and conclude that price dominates quantity at least in the case of multiplicative uncertainty; DAmato and Dijkstra (2010) investigate in the standard Weitzman model the consequences of multiplicative uncertainty. Similar comparisons applied to global warming are made in Pizer (1999), Newell and Pizer (2003), while Pizer (2002) recommends to combine both instruments over time. Recently, Strand (2010b,c) considers in a related but static framework carbon taxes versus cap and trade policies focussing on the bloc of importers that enact a climate policy assuming in all cases price setting oil exporters. Early and also static investigations in a strategic context are Karp (1988) and Wirl (1993).

The following analysis uses a dynamic stock externality game between energy consumers and fossil fuel producers in order to compare the strategic implications of price or quantity instruments applied on the demand as well as on the supply side. Strategic reactions to environmental policies underlie also Sinn's (2008) green paradox: future environmental policies can increase today's extraction of fossil fuels and thus pollution, at least transiently. This argument rests on the assumption of a finite resource stock that will be exploited no matter what happens, but the green paradox does not depend on this premise, see e.g. van der Ploeg and Withagen (2009).

Although the following investigation is theoretical, it is motivated by real world interactions between non-competitive fossil fuel producers (the Organization of Petroleum Exporting Countries, OPEC, and a not unlikely GASPEC in the future) and energy consumers. Of course, energy consumers (or respectively their governments) must be cartelized too in order to have a leverage against supply cartels. In fact, the International Energy Agency (IEA) was founded in 1974 as a response to OPEC (itself founded 50 years ago in 1960) and the first oil price shock in November 1973 with the objective to protect the consumers' interests; how much it did this so far, is an open question. The consumer cartel cares, in contrast to the suppliers, about the damages from global warming. Furthermore, the strategic instruments - price or quantity - have changed during the recent decades on both sides of the energy market. OPEC used price strategies up to 1985 but pursues a quantity policy since 1986. More precisely OPEC's (biannual) Conference

announced and fixed the ‘official selling price’¹ for its reference crude ‘Arab Light’ until 1985, but announces since 1986 production quotas letting the market clear for the price. On the demand side oil, products are taxed heavily in most parts of the worlds and this since a long time. Indeed, the much criticized transport sector is paying some kind of Pigouvian taxes - internalizing its external costs and the costs for the public good of roads and other infrastructure - since many decades while no tax was levied on most other kinds of externalities including fuels and electricity until recently. For example, revenues from taxing petroleum amount to around 10% of all tax revenues in Germany or about to a quarter of (all) income taxes. Even in a year of record high oil prices like 2008 OECD consumer governments’ take from a barrel of refined products ranged between 12% in the US and 57% in the U.K. (with major European countries all above 50%, www.opec.org). Today, global warming is conceived as a substantial threat by most industrialized countries governments, in particular since the Obama² presidency and including some other leading politicians, e.g., Rudd in Australia (now gone, partially due to his support for global warming mitigation). To some surprise, governments prefer quantity instruments for global warming mitigation and this after decades of taxing fossil fuels (at least refined products). This switch to the quantity instrument is documented in the case of the European Union running its Emission Trading System (ETS) for carbon permits since 2005 and in the US as well where the Waxman-Markey cap-and-trade bill proposes also the quantity instrument of permits instead of a carbon tax; this long discussed proposal is now dead after the Democrats abandoned their efforts to limit emissions (*The Economist*, July 31st, 2010, p32-33). Finally, the alternative of regulating producers is infeasible - except for instruments like war and terror - if the crucial fossil fuel producers (OPEC, Russia) are governments of foreign countries.

Formally, the paper departs from the asymmetric differential game in Wirl (1994) solved for the Nash equilibrium in Markov strategies for fossil fuel

¹Before nationalization, the OPEC revenues consisted of taxes based on ‘posted prices’ plus royalties.

²Two quotes suffice to make the point, “My presidency will mark a new chapter in America’s leadership on climate change that will strengthen our security and create millions of new jobs in the process” in the *The New York Times* of November 18, 2008, and climate change speech at the UN, September 2009: “Our generation’s response to this challenge will be judged by history, for if we fail to meet it – boldly, swiftly, and together – we risk consigning future generations to an irreversible catastrophe.”

price and carbon tax; Tahvonen (1996) introduces a first mover advantage by the government, again assuming price instruments on both sides, and Wirl (2007) analysis a stochastic version of this game. The simplification of an antagonistic two-sided world ignores many of the real world constraints yet this choice is a deliberate abstraction in order to reveal key strategic elements, which are the aim of this paper. However, the choice of a dynamic game is mandatory because global warming is a stock externality of emissions of greenhouse gases in particular of carbon dioxide, which is harmless per se (otherwise we could not enjoy a glass of champagne).

Loosely related is also the question how a resource importing monopsony can lower the competitive price by committing (or not) to a consumption path, first Maskin and Newbery (1990) and then Hörner and Kamien (2004) pointing out the formal equivalence with the durable good monopoly; related are also the works of Im (2002) and Strand (2010a) on taxing non-renewable resources. Liski and Montero (2009a) departs from Hörner and Kamien (2004) but require subgame perfection since commitment strategies are implausible in particular if they lack time consistency (a similar issue arises with storable permits, Liski and Montero (2009b)); Bosetti and Victor (2010) stress the importance of credible policies. Therefore the analysis in this papers is also restricted to subgame, more precisely Markov, perfect equilibria and thus the analysis of open loop solutions is entirely eschewed.

Wirl (1994) and all other closely related papers - e.g., Tahvonen (1996), Rubio and Esriche (2001), Liski and Tahvonen (2004) - derive equilibria in Markov strategies of a dynamic game between a benevolent government of consumers and a cartel of fossil fuel suppliers when both play in prices, i.e., the government charges emission taxes. However strategic players can choose quantities instead of prices if it advances their position. Therefore, the framework in Wirl (1994) is extended to investigate how quantities - permits issued by the government and a Cournot playing supply cartel - affect the intertemporal outcomes and to whose advantage. Another sensible extension - assuming that one party is able to commit in the shortrun and thus announces its action just prior to the opponent's action - is also explored.

What are likely outcomes considering these different combinations of instruments? It turns out that quantities are poor instruments, in particular if both choose them, because then an equilibrium (simultaneous moves or first mover advantages by either party) in Markov strategies with positive emissions does not exist. Therefore the current choice of quantities as strategic variables should not persist if the issue of global warming and the strategic

interactions between consumer governments and fossil fuel producers intensify. In particular the strategy of permits is a bad choice in addition to the non-existence result since it eliminates rents from a first mover advantage that are emphasized in Tahvonen (1996) and Liski and Tahvonen (2004) for the tax strategy; similarly for the supply side, choosing a quantity policy surrenders profits if carbon emissions are taxed. In short, the price policies are the undominated strategies on both sides of the market.

The paper is organized as follows. The framework, which considers consumers, fossil fuel suppliers, global warming, and a consumer government, is introduced in Section 2. The equilibria are derived in Section 3 for the four different combinations of price and quantity strategies considering first simultaneous move and then first mover advantages restricted to the short run. The different outcomes are compared and summarized in the final section.

2 Model

The following framework is an adaptation of Wirl (1994), Tahvonen (1996), Rubio and Esriche (2001), and Liski and Tahvonen (2004). It considers three players, consumers, fossil fuel supplier and consumer government of whom consumers are passive and the other two act strategically. The payoffs are affected by a stock externality, which demands a dynamic analysis and thus a differential game framework is applied.

2.1 Consumers

Competitive consumers enjoy the surplus,

$$U(q) = \left(q - \frac{q^2}{2} \right) - Pq, \quad (1)$$

from consuming fossil fuels of the amount (q) . This surplus consists of the normalized linear-quadratic utility between the brackets minus the expenses where P is the final price paid by consumers. As a consequence, inverse demand (or marginal willingness to pay) equals,

$$U' = 1 - q = P. \quad (2)$$

2.2 Global Warming

Consumption of fossil fuels leads to CO2 emissions that accumulate in the atmosphere

$$\dot{X} = q, \quad X(0) = 0. \quad (3)$$

This accounting ignores depreciation (anyway small given the large time constant that a ton of CO2 remains in the atmosphere) and resource constraints (or respectively, global warming determines the ultimate use of fossil fuels, which rules out the green paradox). Since there is no depreciation, a pristine environment is assumed at the start to exclude trivial outcomes with initial conditions of pollution stocks above the steady state. Measuring the stock externality as well as consumption = emission in terms of their net contribution to global warming, this stock induces convex (for simplicity quadratic) external costs,

$$C(X) = \frac{c}{2} X^2. \quad (4)$$

2.3 Fossil fuel supply

Let p denote the price received by the producers, then any difference,

$$P - p = \tau, \quad (5)$$

corresponds either to the tax or the price for permits. Assuming that the producers do not care about the external costs, their objective is to maximize their net present value of profits using the discount rate $r > 0$, and $V(X)$ denotes the corresponding value function,

$$V(X) := \max \int_0^\infty e^{-rt} p(t) q(t) dt. \quad (6)$$

2.4 Consumer Government

The government maximizes the net present value (using the same discount rate $r > 0$) of their consumers' surplus minus the external costs from global warming, i.e.,

$$W(X) := \max \int_0^\infty e^{-rt} \left[q(t) - \frac{q^2(t)}{2} - p(t) q(t) - C(X(t)) \right] dt, \quad (7)$$

subject to the dynamic constraint (3), the suppliers decisions, and market clearing (2). The objective (7) assumes implicitly that tax revenues are returned to consumers as lump sum and similarly for the proceeds from permit sales, because the difference between final consumer and wellhead price constitutes a transfer between consumers that does not count. $W(X)$ denotes the government's value function and of course W as well the supplier's value function depend on the choice of instruments (and equilibrium) and thus differ across the different cases which should be clear without any further indexation.

3 Markov perfect equilibria (MPE)

The analysis is restricted to equilibria in Markov strategies considering first simultaneous move Nash equilibria and then the consequences of first mover advantages. However, all first mover advantages are short run, i.e., one player may announce the strategy in period t before the opponent, of course taking the opponent's reaction into account, compare Basar and Olsder (1982) and Dockner et al. (2000); the extension to announce the entire strategy ahead of the opponent (global Stackelberg solution) is left for future research.

3.1 Prices versus taxes

3.1.1 Simultaneous moves

The Hamilton-Jacobi-Bellman equations (short HJB) implied for the price-tax game,

$$rV = \max_p \{ (1 - p - \tau) (p + V') \}, \quad (8)$$

$$rW = \max_{\tau} \left\{ \frac{(1 - p)^2}{2} - \frac{\tau^2}{2} - \frac{c}{2} X^2 + W' (1 - q - \tau) \right\}. \quad (9)$$

In addition solutions of these two functional equations must satisfy the conditions of no arbitrage (also known as value matching) and of smooth pasting at the point in the state space at which emissions stop. Due to no depreciation, the stopping level is identical to the steady state, denoted by X_{∞} , i.e.,

$$x = 0 \quad \forall X > X_{\infty},$$

hence the corresponding value matching conditions are,

$$V(X_\infty) = 0, \quad (10)$$

$$W(X_\infty) = -\frac{c}{2r}X_\infty^2, \quad (11)$$

since the cartel's value function must be zero in this domain of no emissions = sales and the government's value function is the aggregate net present value of damages. Smooth pasting requires in addition continuity of the derivatives across the stopping and thus at the steady state level,

$$V'(X_\infty) = 0, \quad (12)$$

$$W'(X_\infty) = -\frac{cX_\infty}{r}. \quad (13)$$

Although this game is solved in Wirl (1994) its solution is quickly repeated in order to sketch the crucial solution technique of these asymmetric differential games. Substituting the Nash strategies (from simultaneously maximizing both right hand sides in (8) and (9)),

$$\begin{aligned} p = \frac{1-V'+W'}{2} \\ \tau = -W' \end{aligned} \implies P = p + \tau = \frac{1 - (V' + W')}{2}, \quad (14)$$

into (8) and (9) yields,

$$rV = \frac{(1 + (V' + W'))^2}{4}, \quad (15)$$

$$rW = \frac{(1 + (V' + W'))^2}{8} - \frac{c}{2}X^2. \quad (16)$$

The right hand sides depend only on the sum $(V' + W')$. Therefore, adding up yields a functional equation for the 'pseudo'-value function Z ,

$$rZ = \frac{3(1 + Z')^2}{8} - \frac{c}{2}X^2, \quad Z := V + W, \quad (17)$$

which simplifies the derivation. Guessing a quadratic solution, $Z = z_0 + z_1X + \frac{z_2}{2}X^2$, and comparing coefficients and choosing the root that implies a stable strategy (here for P),

$$\begin{aligned} z_2 &= \frac{2}{3} \left(r - \sqrt{r^2 + 3c} \right), \\ z_1 &= \frac{3z_2}{4r - 3z_2}, \end{aligned}$$

determines the solution of the functional equation (17); then (15) and (16) are solved by similar means (i.e., assuming quadratic V and W , substituting the above solution for Z , and comparing coefficients). This yields the result summarized below.

Proposition 1: *The explicit solution of the strategies*

$$P = \frac{1 - (z_1 + z_2 X)}{2} = 1 + \frac{(r - \sqrt{r^2 + 3c})}{3} \left(X - \frac{r}{c}\right), \quad (18)$$

$$p = \frac{r(r - \sqrt{r^2 + 3c}) - 6c}{9r} \left(X - \frac{r}{c}\right) \quad (19)$$

$$\tau = 1 + \frac{6c + 2r(\sqrt{r^2 + 3c} - r)}{9r} \left(X - \frac{r}{c}\right) \quad (20)$$

implies that consumer price and tax increase up to the choke price, P and $\tau \rightarrow 1$, which is asymptotically reached at the steady state³,

$$X_\infty = \frac{r}{c}. \quad (21)$$

Therefore the steady state of this non-cooperative dynamic game is the socially efficient one. However, the intertemporal allocation is not efficient, more precisely, emissions are too conservationist, because the socially optimal consumer price is lower for any $X < X_\infty$. The producer price declines monotonically to zero (again reached at the steady state) yet may exceed the usual monopoly price (equal to $1/2$) due to preemption. In fact, larger external costs increase the initial price and possibly also the preemption domain (i.e., where $p > 1/2$).

Remark: Observe that the conditions of value matching and smooth pasting are satisfied without being used in the derivation of the linear strategies (this applies in sequel as well). However, these conditions turn out as useful if only nonlinear strategies allowed for emissions. Furthermore note that the steady state as well as the split of P into wellhead price and tax at this steady state, $p(X_\infty) = 0$, $P(X_\infty) = \tau(X_\infty) = 1$, follows immediately from the value matching and smooth pasting conditions and (15) and (16).

An interesting feature is that the externality induces the monopoly to preempt the tax. More precisely, the monopoly charges at any point in time

³This steady state follows from the economically intuitive condition of equating the marginal benefit from consuming/emitting the last unit ($U'(0) = 1$) to the induced net present value of marginal damage, cX/r .

the price above the static Nash response, $p > \frac{1-\tau}{2}$ (weak preemption), and this price can even exceed the (static) monopoly price, $p > 1/2$ for some time (strong preemption). This domain of strong preemption can be obtained in explicit form ($p > 1/2$ for $X < \tilde{X}$ as given in Fig. 1). Fig. 1 plots the domains - absolute and relative - of preemption with respect to the external costs for numerical examples: preemption domain declines monotonically in absolute terms (no surprise since also the steady state declines, but is nonmonotonic when varying the discount rate) but increases for the relative measure with the limit $\lim_{c \rightarrow \infty} (\tilde{X}/X_\infty) = 1/4$, i.e., a quarter of the total sales will be sold above usual monopoly price in attempt to deter and to delay taxation.

[Insert Fig. 1 approximately here]

3.1.2 Short run commitments

The above analysis of the simultaneous moves of the players is now extended by giving one of the players the ability of shortrun commitment, i.e., to announce his policy in each period t just ahead of its rival. Allowing for such intraperiod first mover advantages, the results have to be modified in the following way:

1. Rendering the ability of short run commitment to the supplier reproduces the above Nash outcome, because the government's action is not affected directly by the cartel's price announcement because of the tax is set equal to the marginal damage ('Pigouvian'), $\tau = -W'$, according to (14).
2. Letting the government choose the tax $\tau(t)$ at any point in time ahead of the producers yields however a different outcome. Substituting the cartel's price reaction from (14) into the right hand side of (9) and then maximizing with respect to τ yields,

$$\tau = \frac{1 + V' - 2W'}{3},$$

which implies in turn

$$p = \frac{1 + W' - 2V'}{3}.$$

Substitution of these strategies into (8) and (9) yields a pair of functional equations that corresponds to the game of a tax setting government (including its ability to short run commitment) and a quantity setting producer. Hence, the solutions are identical and the corresponding result is given in the following section in order to include similar outcomes with in one proposition (Proposition 2).

3.2 Taxes and production quotas

3.2.1 Simultaneous moves

Assuming Cournot behavior of the fossil fuel cartel, the objectives of the two players are,

$$\max_{\tau} \int_0^{\infty} e^{-rt} \left[q - \frac{q^2}{2} - (1 - q - \tau) q - C(X) \right] dt, \quad (22)$$

$$\max_q \int_0^{\infty} e^{-rt} (1 - q - \tau) q dt. \quad (23)$$

Hence, the Hamilton-Jacobi-Bellman equations are,

$$rW = \max_{\tau} \left\{ q\tau + \frac{q^2}{2} - C(X) + W'q \right\}, \quad (24)$$

$$rV = \max_q \{ (1 - q - \tau) q + V'q \}. \quad (25)$$

Given the linearity of the objectives with respect to τ (in particular on the right hand side of (24)), no interior Nash equilibrium can exist.

3.2.2 Short run commitments

Allowing the cartel the first move at each period t does not alter this conclusion. However, rendering the ability of shortrun commitment to the government allows for an interior policy because substituting the supply cartel's Nash-Cournot reaction,

$$q^N = \frac{1 - \tau + V'}{2}, \quad (26)$$

into the rhs of (24) yields,

$$rW = \max_{\tau} \left\{ \frac{W'(1 + V' - \tau)}{2} + \frac{(1 + V' + \tau)^2}{8} - \frac{\tau^2}{2} + \frac{q^2}{2} - C(X) \right\}. \quad (27)$$

Maximization of the right hand side yields the tax,

$$\tau = \frac{1 + V' - 2W'}{3}, \quad (28)$$

which is identical to the tax derived for a government having the first move (at any period t) in the price versus tax game as mentioned above. This tax (28) implies for the cartel's output,

$$q = \frac{1 + V' + W'}{3}. \quad (29)$$

As a consequence, the consumer price (and thus the emission) depends only on the sum of the value functions' derivatives as above in the price-tax game,

$$P = \frac{2 - (V' + W')}{3}. \quad (30)$$

Substitution of the strategies (28) and (29) into the right hand sides of the HJB-equations (25) and (27) yields that not only the final consumer price but also that the right hand sides in (25) and (27) depend only on this aggregate Z introduced in (17),

$$rV = \frac{(1 + Z')^2}{9}, \quad (31)$$

$$rW = \frac{(1 + Z')^2}{6} - \frac{c}{2}X^2. \quad (32)$$

Therefore, adding up leads again to a single functional equation

$$rZ = \frac{5(1 + Z')^2}{18} - \frac{c}{2}X^2, \quad (33)$$

with the following coefficients for the quadratic solution of this functional equation,

$$\begin{aligned} z_2 &= \frac{3}{10} \left(3r - \sqrt{9r^2 + 20c} \right), \\ z_1 &= \frac{5z_2}{9r - 5z_2}. \end{aligned}$$

Proceeding as above allows to calculate the players' strategies explicitly.

Proposition 2: *An intertemporal and interior equilibrium in Markov strategies for a quantity setting supply cartel and a taxing government requires short run commitment (of the first moving) government when setting the intraperiod tax. The equilibrium is characterized by the same stationary stock of pollution (21) and the same qualitative picture as in Proposition 1 emerges: consumer prices,*

$$P = \frac{2 - z_1 - z_2 X}{3} = 1 + \frac{\sqrt{9r^2 + 20c} - 3r}{10} \left(X - \frac{r}{c} \right) \quad (34)$$

and taxes

$$\tau = 1 + \frac{10c/r + 2(\sqrt{9r^2 + 20c} - 3r)}{25} \left(X - \frac{r}{c} \right) \quad (35)$$

are increasing to the choke price, and the producer price,

$$p = \frac{\sqrt{9r^2 + 20c} - (20c/r + 3r)}{50} \left(X - \frac{r}{c} \right) \quad (36)$$

is declining to zero. However, the tax is higher (conditional on X) and the producer price is lower - ruling out any kind of preemption, $p < 1/2$ - but such that final consumer prices are increased thereby lowering emission. This difference compared with Proposition 1 is due to the additional scope of rent acquisition by the government through higher taxes. Finally, this outcome is identical to the case if a taxing government has the first move (at any period t) in the price versus tax game as mentioned above.

Proof: Above derivation; for the remaining arithmetical claims see Appendix.

Fig. 2 shows a numerical example that compares the outcomes (in the \$-domain) for the two different scenarios addressed in Propositions 1 and 2. The nearly identical consumer price but the substantial difference in well-head prices and thus also in taxes highlights the rent accrued by a taxing government having the first move independent whether the suppliers choose a quantity or price strategy.

[Insert Fig. 2 approximately here]

If the supply cartel chooses quantity as its strategic variable - and this is what OPEC does since 1986 - it will offer its opponent, a welfare maximizing consumer government, a strategic advantage. Therefore, given this

disadvantage, a rational monopoly should prefer the price strategy rendering a quantity strategy as unlikely in the case of an emerging rent contest about carbon taxes between OPEC and IEA. OPEC used indeed price increases strategically to deter taxation, e.g.: in 1986, Saudi Arabia was determined to punish quota cheating OPEC members, in particular Nigeria, yet restored prices quickly once US politicians threatened in the presence of low oil prices with an oil import fee. *The Economist* (1992) explains the oil price increase of almost \$4 per barrel prior to the Earth Summit in Rio de Janeiro between March and June 1992 by strategic Saudi Arabian actions, ‘to see the price rise by \$3 a barrel to match the effect of the first step in the EC’s new carbon-tax plan.’

The high oil prices during 2008 (although not set deliberately given OPEC’s quota policy) proved to be a viable deterrent to introduce carbon taxes: some countries withdrew their plans on future carbon taxes and others even reduced fuel taxes when facing angry voters (e.g. in France for fisher and farmers), while Kenneth Rogoff⁴ argues for taxing carbon, because "either we raise the price of oil, or OPEC will." Therefore, the 2008 experience questions whether consuming governments are able or willing to set high taxes in advance in order to lower import prices for fuels, in particular if import prices are high already.

3.3 Permits and price setting cartel

3.3.1 Simultaneous moves

Assuming that the government issues the amount $\bar{q}$ of permits implies that either the volume of permits or the price charged by the cartel determines final demand and thus emission,

$$q = \min \{ \bar{q}, 1 - p \} , \quad (37)$$

i.e., not all permits will be used if the cartel’s price is too high. The objective of the government issuing permits (grandfathered) is:

$$\max_{\bar{q}} \int_0^{\infty} e^{-rt} \left[q - \frac{q^2}{2} - pq - C \right] \text{ s.t. (3) and (37)}. \quad (38)$$

⁴quoted from *The Economist*, "A Survey of Oil", on p4, April 30th 2005.

What is the optimal amount of permits given the supply price p ? It is clearly suboptimal (or at least ineffective) to issue more permits than $1 - p$, the market clearing at the wellhead price leaving no margin for permit prices. Hence, the amount of permits equals or falls short of the market demand, $\bar{q} \leq 1 - p$, the latter if the supplier chooses a low price and the government feels that the resulting demand adds too much to the stock externality. Therefore, maximizing the rhs of the HJB equation,

$$rW = \max_{0 \leq \bar{q} \leq 1-p} \left\{ q - \frac{q^2}{2} - pq - C(X) + W'q \right\}, \quad (39)$$

yields as interior solution,

$$\bar{q} = 1 - p + W'.$$

Since $W' < 0$, the upper bound cannot bind, the governments optimal allocation of permits is,

$$\bar{q} = \begin{cases} 1 - p + W' & \geq 0 \\ 0 & < 0 \end{cases} \quad \text{if } 1 - p + W' \begin{cases} \geq 0 \\ < 0 \end{cases} \quad (40)$$

The HJB-equation of the price setting cartel facing the permit constraint $\bar{q}$ is

$$rV = \max_p \{ p \min \{ \bar{q}, 1 - p \} + V' \min \{ \bar{q}, 1 - p \} \} \quad (41)$$

$$= \max_{p \geq 1 - \bar{q}} \{ p(1 - p) + V(1 - p) \}. \quad (42)$$

Maximizing the right hand side implies the price,

$$p = \begin{cases} \frac{1-V'}{2} & \geq \frac{1+V'}{2} \\ 1 - \bar{q} & < \frac{1+V'}{2} \end{cases} \quad (43)$$

which is shown in an inverted form (as a reaction function) in Fig. 3. In words, the cartel will charge at least $p = \frac{1-V'}{2}$ and the market clearing price, $p = 1 - \bar{q}$, for less permits ($\bar{q} < 1 - \frac{1-V'}{2}$). The reason is that suppliers facing the permit quota will choose the market clearing price in order to accrue the entire rent by driving the permit price to zero. A consequence of these Nash

reactions (40) and (43) is that no sensible, i.e., interior $q > 0$ simultaneous move Nash equilibrium exists, see Fig. 3, since $q = 0$ is the only point of intersection. Therefore, the alternatives of short run commitment by either player are considered in two subsections.

Proposition 3: *If the consumers' government issues permits and the supply cartel sets prices, then only the boundary solution,*

$$p = 1 \text{ and } \bar{q} = 0, \quad (44)$$

qualifies as a Nash equilibrium in Markov strategies.

[Insert Fig. 3 approximately here]

3.3.2 Cartel moves first

Assume that the price setting cartel moves first. The government's Nash reaction is to issue permits below the market clearing level,

$$\bar{q} = 1 - p + W'. \quad (45)$$

Therefore the HJB of the at each point in time first moving cartel must satisfy,

$$\begin{aligned} rV &= \max_p \{(1 - p + W')(p + V')\} \\ \implies p &= \frac{1 + W' - V'}{2}. \end{aligned} \quad (46)$$

Substitution into the government's Nash reaction gives,

$$q = \bar{q} = \frac{1 + (V' + W')}{2}, \quad (47)$$

and (46) and (47) into the HJB equations (39) and (41) implies,

$$rV = \frac{(1 + (V' + W'))^2}{4}, \quad (48)$$

$$rW = \frac{(1 + (V' + W'))^2}{8} - \frac{c}{2}X^2. \quad (49)$$

These functional equations are identical to those obtained under the assumption that price and tax are the policy instruments. As a consequence, the

same allocation as in Proposition 1 results with the difference that no tax revenues are accrued. Since the cartel moves first, its price will not clear the market, $p < 1 - \bar{q}$, because the government allocates permits below the market clearing price. This implies a positive permit price such that the corresponding revenues cannot be accrued by the supply cartel and thus remain within consuming countries as the tax revenues in Section 3.1, either as the proceeds from auctioning permits or as a rent for consumers if permits are grandfathered.

Proposition 4: *A government issuing permits facing a price setting cartel, which has the first move, reproduces the outcome in Proposition 1. The difference is that the permit price covers the wedge between producer's and consumer's price instead of the tax (equal to the tax given in Proposition 1), because the government will issue permits below the market clearing level given the cartel's supply price (determined as outlined in Proposition 1).*

3.3.3 Government moves first

From the above analysis it seems that the incentives for a cartel to move first are weak, because following the government's issuance of permits allows a price setting cartel to accrue all rents by charging the price that clears the market given the number of permits. However, in spite of this apparent second mover advantage, the cartel has a strong incentive to move first, because the first moving government takes these reactions into account.

Assume that the government moves first and distributes $\bar{q}$ permits. Since the cartel will charge at least $p = \frac{1-V'}{2}$, issuing permits above the corresponding market clearing level, $\bar{q} > 1 - \frac{1-V'}{2} = \frac{1+V'}{2}$, is without any consequence, i.e.,

$$q = \min \left\{ \bar{q}, \frac{1+V'}{2} \right\}. \quad (50)$$

Substituting the cartel's Nash reaction given in (43) and sketched in Fig. 3 into the right hand side of the HJB of a permit issuing government and accounting for the above constraint (50) on emissions, gives

$$rW = \max_{0 \leq \bar{q} \leq \frac{1+V'}{2}} \left\{ \bar{q} - \frac{\bar{q}^2}{2} - p(\bar{q}) \bar{q} - C + W' \bar{q} \right\}. \quad (51)$$

The cartel chooses the market clearing, $p(\bar{q}) = 1 - \bar{q}$, in its attempt to accrue

the maximum of rent. The resulting consumer surplus (U),

$$rW = \max_{0 \leq \bar{q} \leq \frac{1+V'}{2}} \left\{ \frac{\bar{q}^2}{2} - C + W'\bar{q} \right\},$$

is convex in the government's instrument ($\bar{q}$). Therefore, the maximum on the right hand side of (51) must be at a boundary (in spite of the interior 'Pigouvian solution', $\bar{q} = -W'$): either no permits or permits equal to the cartel's maximum supply,

$$\bar{q} = \begin{cases} \frac{1+V'}{2} & \text{if } \frac{1+V'}{2} > -2W' \\ 0 & \text{if } \frac{1+V'}{2} \leq -2W' \end{cases} \quad (52)$$

The choice between these alternatives depends on the maximum volume supplied by the cartel (and thus on V') and on the shadow costs from global warming ($W' < 0$). The first inequality in (52) holds if the social surplus from emissions (valuing the damage at $-W'$ per unit emission) is still positive even if the suppliers accrue all potential tax rents. If this is the case, the number permits equals the market clearing demand at the cartel's price,

$$\bar{q} = q = \frac{1+V'}{2}.$$

This hypothesis of positive emissions implies the following pair of functional equations,

$$rW = \frac{1+V'}{2} \left(\frac{1+V'}{4} + W' \right) - C(X), \quad (53)$$

$$rV = \left(\frac{1+V'}{2} \right)^2. \quad (54)$$

Proposition 5: *There exists no equilibrium in Markov strategies supporting positive quantities, neither in singular = linear nor in non-linear strategies. Hence, $\bar{q} = 0$, $p = 1$, results as the unique equilibrium as in the Nash case (characterized in Proposition 3).*

Proof: No pair of solution curves of () and () qualifies for a value function since they cannot satisfy the boundary conditions (10) - (13); for details see Appendix.

3.4 Emission permits and production quotas

3.4.1 Simultaneous moves

Letting both sides play in quantities seems at first sight impossible because both parties cannot fix the quantity at the same time. However, each party may overwrite the other's choice in particular if one party moves first. Moreover, this seems to be the future game in town: EU-countries issue permits since 2005 (as mentioned, U.S. were expected to follow with the Obama presidency) and OPEC determines output quotas at its biannual Conference. For example, the suppliers plan to sell Q yet the government deems this too high and issues less permits, $\bar{q} < Q$. Conversely, the government may issue a lot of permits yet the suppliers find it in their interest to cut output below the number of permits, $Q < \bar{q}$. For example consider negligible damage, $c \rightarrow 0$, then $\bar{q} \rightarrow 1$ would be socially optimal yet the cartel will deliver only the half of it, $Q \rightarrow 1/2$. Therefore,

$$q = \min \{ \bar{q}, Q \} . \quad (55)$$

The HJB-equation of a government issuing permits (grandfathered) is

$$rW = \max_{\bar{q}} \left\{ q - \frac{q^2}{2} - (1 - q) q - C + W' q \right\} , \quad (56)$$

and the counterpart of the supplier is,

$$rV = \max_Q \{ (1 - q) q + V' q \} , \quad (57)$$

in both cases accounting for (55).

The government's Nash reaction if facing the supply strategy Q allows for two cases: $\bar{q} > Q$ in which case the government's choice is irrelevant and thus it may choose, $\bar{q} = Q$, or it is determinant, $\bar{q} < Q$. Therefore,

$$\begin{aligned} rW &= \max_{\bar{q} \leq Q} \left\{ \bar{q} - \frac{\bar{q}^2}{2} - (1 - \bar{q}) \bar{q} - C + W' \bar{q} \right\} \\ &= \max_{\bar{q} \leq Q} \left\{ \frac{\bar{q}^2}{2} - C(X) + W' \bar{q} \right\} . \end{aligned} \quad (58)$$

This is as in the previous case of a government announcing permits and the supply cartel choosing price (as a follower). Hence, the optimum must be

again at the boundaries as outlined in the previous section with the same corresponding switching level,

$$\bar{q} = \begin{cases} Q & \geq \\ 0 & < \end{cases} \text{ if } Q + 2W' > 0. \quad (59)$$

As a consequence, the government will again issue permits only if $Q + 2W' > 0$, i.e., for either sufficiently large volumes supplied by the cartel or small external costs.

Similarly, the supplier's choice of $Q > \bar{q}$ is irrelevant and if the choice of Q is determinant, then

$$rV = \max_{Q \leq \bar{q}} \{(1 - Q)Q + V'Q\}. \quad (60)$$

Hence, the supplier's decision is,

$$Q = \begin{cases} \frac{1+V'}{2} & \geq \\ \bar{q} & < \end{cases} \text{ if } \bar{q} \geq \frac{1+V'}{2} \quad (61)$$

and thus interior only if it is determinate. Therefore, a shortrun equilibrium must satisfy,

$$0 \leq \bar{q} \leq \frac{1+V'}{2}, \quad (62)$$

because the cartel will reduce its output to the level given on the right hand side if a larger volume $\bar{q}$ of permits is offered.

For any amount of permits satisfying the above inequality (62) strictly, the cartel will just produce the corresponding level, $Q = \bar{q}$. The other player, the government, will only issue permits, if $Q + 2W' > 0$, i.e., for sufficiently high volumes supplied by the cartel, otherwise it will issue no permits at all due to (61). Therefore, at an equilibrium with positive emissions,

$$Q = \frac{1+V'}{2} = \bar{q} > -2W'. \quad (63)$$

Otherwise, the equilibrium must be a corner solution,

$$Q = \bar{q} = 0. \quad (64)$$

Fig. 4 plots the corresponding Nash reaction functions if both players choose quantity as their strategy, i.e. (59) and (61), and shows the implied Nash equilibria, either (63) or (64). Substituting the positive emissions from (63) yields, (54) and (53) as in the previous case of a government issuing permits before the price setting cartel chooses its price. Since the corresponding solutions of these two functional equations cannot constitute the value functions of such a game, no interior equilibrium in Markov strategies exists.

[Insert Fig. 4 approximately here]

Proposition 6: *There exists no Nash equilibrium in Markov strategies with positive emissions if both players choose quantities even if allowing for nonlinear strategies. Therefore, $\bar{q} = 0$, $p = 1$, is the unique equilibrium.*

3.4.2 Short run commitments

Substituting the government's Nash reaction (59) into the cartel's optimization problem,

$$rV = \max_Q \{(1 - Q)Q + V'Q\},$$

implies the same supply policy (61) as in the above Nash game and thus the same intertemporal allocation.

Letting the government move first and substituting the cartel's reaction (61) into the government's HJB-equation yields also (58). Hence, the same convex maximand implies the same boundary strategies, $\bar{q} = Q = \frac{1+V'}{2}$ or $\bar{q} = 0$, and thus the same intertemporal allocation.

Proposition 7: *Assuming the ability of either party to move first ends up in both cases in the inefficient Nash equilibrium characterized in Proposition 6.*

4 Comparison and summary

This paper considered the strategic implications of price and quantity instruments in a game between a consumer government caring about global warming (a stock externality, which requires a dynamic framework) and profit maximizing fossil fuel supply with both sides being monopolized. The results are illustrated in Table 1. If the government sets a tax while the monopolist simultaneously sets the price, there is a unique Markov perfect equilibrium

(MPE) in linear strategies. In this equilibrium, the tax increases over time, the monopolist's price declines, and the stock of pollution, X , increases and converges to the efficient level, X^* (but at less than the socially optimal speed) written as $X \uparrow X^*$. If, at each moment in time, the monopolist sets the price before the country sets its tax, the outcome is given in the second sub-column of the first column and the outcome is, as indicated, then identical to the simultaneous move case. If the government acts first, there exists also a unique MPE in linear strategies, but the tax is now larger because since this induces the monopolist to reduce its price. Although X converges to X^* as before, it increases more slowly than in the first case since the final consumer price is higher; this difference is indicated by writing $X \nearrow X^*$. The interpretation of the remaining outcomes is then straightforward with the help of Table 1. Note that all three considered outcomes under the assumption that both players choose quantities end up in the corner equilibrium (even after allowing for nonlinear Markov strategies) and only one interior equilibrium exists when the monopoly fixes output, namely if the government announces its tax ahead of the monopoly's choice. As a consequence, if the monopoly prefers to play in quantities, then it would like to surrender the ability of shortrun commitment to the consumer government, if it sets prices then shortrun commitment is beneficial for the monopoly.

Table 1: Comparison of the results

	M - price			M - quota		
	sim	M 1 st	G 1 st	sim.	M 1 st	G 1 st
G - tax	$X \uparrow X^*$	$X \uparrow X^*$	$X \nearrow X^*$	$X = 0$	$X = 0$	$X \nearrow X^*$
G - quota	$X \uparrow X^*$	$X \uparrow X^*$	$X \nearrow X^*$	$X = 0$	$X = 0$	$X = 0$

M = monopoly, G = consumer government, sim = simultaneous moves
 $\uparrow$ convergence, $\nearrow$ slower convergence due to higher consumer price P ,
 $X^* = 1^{st}$ best.

Although most of the related papers do not accept commitment strategies and require quite plausibly subgame perfection, they take the choice of either the price or the quantity instrument as given (or even as equivalent) without considering the strategic implications of the alternative instrument. However parties do choose and even switch between them. Therefore given the different combinations analyzed above, what are the likely choices of these instruments? Starting with the supply side, price and quantity are

equivalent instruments without externalities at least in the Nash setting (or no carbon policy). Using the price-tax Nash equilibrium as a benchmark, choosing quantity as the strategic variable - and this is what OPEC does since 1986 after the former price setting regimes (first of tax based prices + royalties and then official selling prices) - renders a strategic advantage to its opponent, the government. Therefore, a rational monopoly will prefer the price strategy in the case of an emerging rent contest between OPEC and IEA over carbon taxes. As a consequence, OPEC should be expected to switch back to pricing if consumer governments tax carbon. Assuming therefore that the supply cartel sets prices, a Nash playing government is indifferent between the price and quantity instrument. However, if the government has the first move in each period, then this ability of short run commitment is only beneficial if it taxes carbon in which cases it can eliminate preemption by the cartel. In fact, a permit strategy is harmful for both players if either the government has the first move or if the supply decides to play in quantities too (and then irrespective who has the first move, or if they move simultaneously in each period). More precisely, no equilibrium with positive emissions exists. The abandoning of cap and trade in the energy bill recently unveiled by the Democratic majority in Senate might quite unintentionally lead to the more efficient alternative of carbon taxes.

Summarizing, quantities are bad choices for both parties (albeit for different reasons) such that prices and taxes are the natural choices in this strategic setup. This is surprising in the light of the fact that today's players seem to prefer quantity strategies with consumer governments eschewing carbon taxes and issuing permits and with OPEC announcing quantities instead of setting prices. Therefore, the model's predictions of a lack of an equilibrium with both players choosing quantities needs reconciliation with reality. There are of course many explanations outside of the model and most of them are presumably political in nature, in particular with respect to the preference for carbon permits over taxes, because increasing fuel taxes are conceived to harm incumbent policy makers.

The explanations below are related to the applied framework in a way that points towards potential extensions. Firstly, and presumably crucial is the fact that fossil fuels are also extensively taxed and this since times when no politician even imagined the global warming externality. In fact, given this long experience it is somehow even a political puzzle why permits were chosen in order to mitigate global warming and to include the so far by and large untaxed segments (power plants, industries) instead of broadening the

energy tax base. In fact the bulk of oil demand is taxed (around 3 quarters in the case of OECD consist of light and middle distillates (BP (2010)), which are taxed) and thus a relatively small volume of total consumption is actually subject to quantity restrictions. Consumer governments' policies of taxes and quotas, taxing one segment of demand and issuing permits to another sector, may provide additional flexibility (in particular to mix them differently over as the already quoted paper Pizer (2002) suggests). However, allowing for both instruments requires to split demand into a segment that is taxed and into another one that faces quotas and it is doubtful whether such splitting makes sense. Secondly, the assumption of completely cartelized supply is significantly off the mark for today's world energy markets even given the strong positions of OPEC, of Russia concerning Europe's natural gas supply, and of a GASPEC (with three countries - Russia, Iran and Qatar - holding more than 50% of the reserves unless shale gas supply exceeds its hyped potential) in the pipeline. Thirdly, the assumption of cartelized consumers seems even less plausible with a toothless IEA and many developing countries even subsidizing fossil fuels. Indeed most of the current incremental demand for fossil fuels comes from countries with no ambition at all to introduce policies in order to mitigate global warming. Including these aspects requires substantial extensions such as: competition in fossil fuel supply if possible accounting for the characteristic of imperfect substitutes (oligopolistic, e.g. in oil (OPEC) and natural gas ('GASPEC'), and the rest may be simplified to competitive fringe suppliers); differences in demand, in particular adding players with either no environmental policy (i.e. passive) or free riding, i.e., entering as additional and active players, etc. Thirdly, political aspects are equally important. For example, OPEC's real decision makers may have (political) objectives beyond maximizing profits (for an example see Wirl (2009)) and similarly consumer governments usually like to get re-elected or are captured by particular interest groups, yet the Leviathan motive as assumed in Dockner and Wirl (1995) and Wirl (1996) can be ruled out for current governments issuing permits for free. Such extensions clearly improve realism but appear not only unsolvable if introduced simultaneously but even if solvable (say numerically) it may bury key strategic insights, which are the aim of this paper. More precisely, accounting for either competitive fringe supply and/or additional demand generated by governments ignorant of the stock externality may then yield an interior equilibrium instead of the corners solutions shown in Table 1, yet the crucial implication - prices dominate the quantity instruments on both sides - seems robust.

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6 Appendix:

6.1 Proof of Proposition 2

6.1.1 Consumer price

Given the same steady state and the same consumer price at the steady state it is sufficient to compare the intercepts. The game in prices and taxes yields,

$$P^a(0) = \frac{r^2 + 3c - \sqrt{r^4 + 3r^2c}}{3c}$$

and for a quantity setting cartel confronted with a first moving and taxing government,

$$P^b(0) = \frac{3r^2 + 10c - \sqrt{9r^4 + 20r^2c}}{10c}.$$

The claim is equivalent to $P^b(0) - P^a(0) > 0$. Cancelling the term c from the denominator, i.e.,

$$\Delta(c) := c(P^b(0) - P^a(0)) = \frac{10\sqrt{r^4 + 3r^2c} - 3\sqrt{9r^4 + 20r^2c} - r^2}{30}$$

Note that $\Delta(0) = 0$ and that

$$\Delta' = \frac{r}{2} \left(\frac{1}{\sqrt{r^2 + 3c}} - \frac{2}{\sqrt{9r^2 + 20c}} \right)$$

is definitely positive, because the bracket is positive iff

$$\sqrt{9r^2 + 20c} > 2\sqrt{r^2 + 3c} \iff 9r^2 + 20c > 4r^2 + 12c$$

6.1.2 Producer price

Similar for the producer prices,

$$p^a(0) = \frac{r^2 + 6c - \sqrt{r^4 + 3r^2c}}{9c}$$

$$p^b(0) = \frac{3r^2 + 20c - \sqrt{9r^4 + 20r^2c}}{50c}.$$

The claim is equivalent to $p^a(0) - p^b(0) > 0$. Cancelling the term c from the denominator, i.e.,

$$\Delta(c) := c(p^a(0) - p^b(0)) = \frac{120c + 23r^2 + 9\sqrt{9r^4 + 20r^2c} - 50\sqrt{r^4 + 3r^2c}}{450}$$

which implies again that $\Delta(0) = 0$ and that the derivative

$$\Delta' = \frac{1}{30} \left(8 + \frac{6r}{\sqrt{9r^2 + 20c}} - \frac{5r}{\sqrt{r^2 + 3c}} \right)$$

is positive, because

$$\begin{aligned} 8 + \frac{6r}{\sqrt{9r^2 + 20c}} - \frac{5r}{\sqrt{r^2 + 3c}} &> 0 \\ &\iff \\ \frac{6}{5} + \frac{8}{5} \sqrt{\frac{9r^2 + 20c}{r^2}} &> \sqrt{\frac{9r^2 + 20c}{r^2 + 3c}}. \end{aligned}$$

6.1.3 No preemption

The claim of no preemption follows also immediately from the explicit solution. Preemption, if taking place at all must apply at $X = 0$, therefore the claim implies,

$$p(0) = \frac{20c + 3r^2 - \sqrt{9r^4 + 20cr^2}}{50c} < \frac{1}{2},$$

where the claimed inequality holds iff,

$$3r^2 - 5c < \sqrt{9r^4 + 20cr^2}. \quad (65)$$

This inequality holds trivially if the lhs is negative. Hence, violation of the claim requires at the minimum,

$$0 < 3r^2 - 5c \implies c < \frac{3}{5}r^2. \quad (66)$$

Assuming indirectly that $p(0) > \frac{1}{2}$, squaring and re-arranging implies $c > 2r^2$. This exceeds the bound (66), renders therefore the left hand side negative in (65) such that inequality (65) holds, which contradicts $p(0) > \frac{1}{2}$. QED.

6.2 Proof of Proposition 5

The singular solution of (54),

$$V = \frac{1}{4r}, \quad (67)$$

cannot be the value function of this game, because it corresponds to a cartel that charges the usual monopoly price, $p = 1/2$ forever and to the government policy of issuing the corresponding volume of permits, $\bar{q} = 1/2$ (reverse order in each period t since the government moves first). This government policy is however not optimal: it will drive the marginal damage beyond any bound (utility is bounded by $3/8$, marginal damage is unbounded) such that sooner or later $W'(X) \leq -1/4$. As a consequence of (52), the government must stop issuing permits (of the volume $\bar{q} = 1/2$) at finite time such that the monopoly's profit are zero from this moment on for all future and so for any pollution stock beyond this threshold. Therefore, (67) cannot be the corresponding value function.

The alternative of nonlinear (or non-singular) solutions of (54) can be sketched in the phase diagram (see Fig. 5) and can be even given in closed form:

$$V(X) = \frac{\left(1 + \varphi\left(-e^{-1+2r(X-k)}\right)\right)^2}{4r}, \quad (68)$$

where k is an integration constant and the function $\varphi(z)$ is the solution of,

$$z = \varphi e^\varphi, \quad \varphi' = \frac{1}{\varphi e^\varphi + e^\varphi} = \frac{\varphi}{(\varphi + 1)z}, \quad (69)$$

which has support for all $z > -1/e$, is increasing due to the implicit function theorem, and $\varphi'(0) = 1$, $\varphi(0) = 0$, $\lim_{z \rightarrow -1/e} \varphi(z) = -1$. The proof follows from differentiating the solution (68) using (69) and simplifying. The corresponding solutions imply interior output levels for $0 > V' > -1$. These supply strategies do not change the government's all or nothing strategy concerning the issuing of permits. This however excludes a continuous phasing out of carbon emissions. Therefore, at this moment of stopping - shown in Fig. 4 when a solution of V' crosses the equality in (52)⁵ - no solution of (54) can satisfy simultaneously the necessary optimality condition for stopping at a finite date: $V = 0$ and $V' = 0$, known as smooth pasting.

⁵Actually, each chosen nonlinear solution of V' implies a different W' but only one corresponding solution is shown in Fig. 4, which is sufficient to make this point.

[Insert Fig. 5 approximately here]

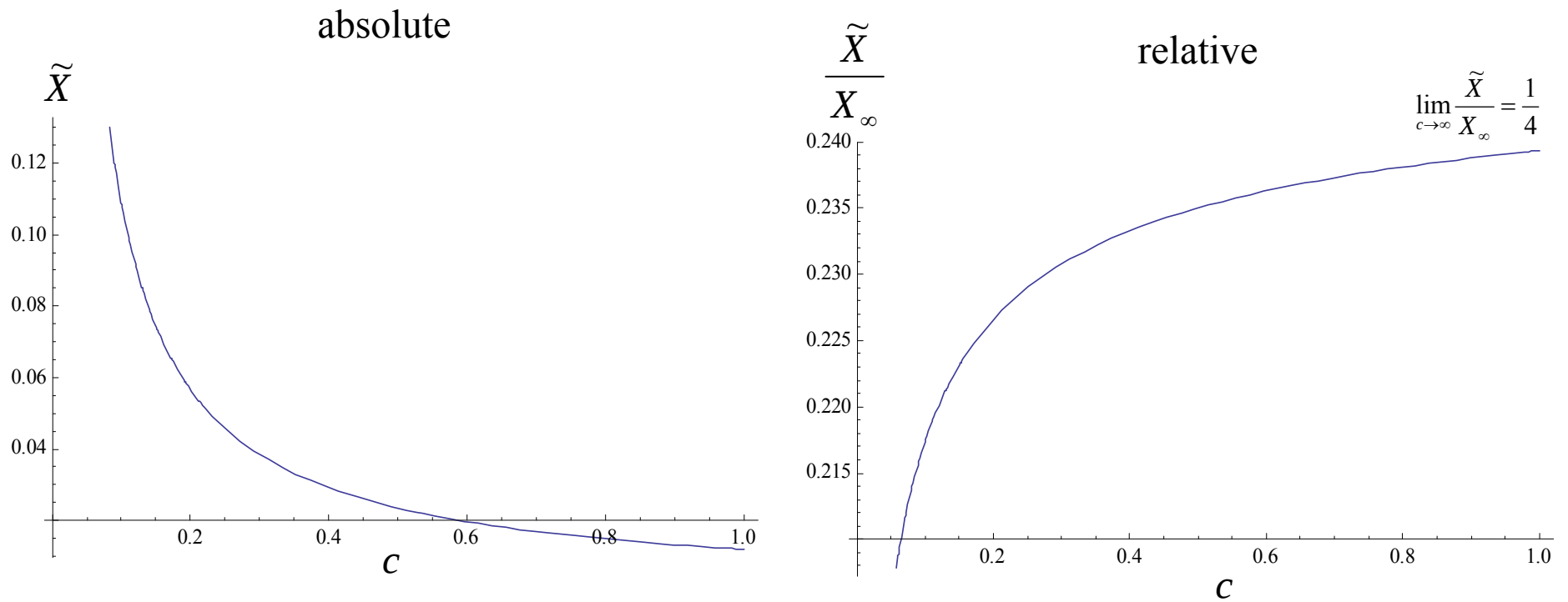


Fig. 1: String preemption domain, $p > \frac{1}{2}$, absolute and relative ($r = 0.05$)

$$\tilde{X} = \frac{r^2 + \frac{3}{2}c - \sqrt{r^4 + 3r^2c}}{r^2 + 6c - \sqrt{r^4 + 3r^2c}} X_\infty$$

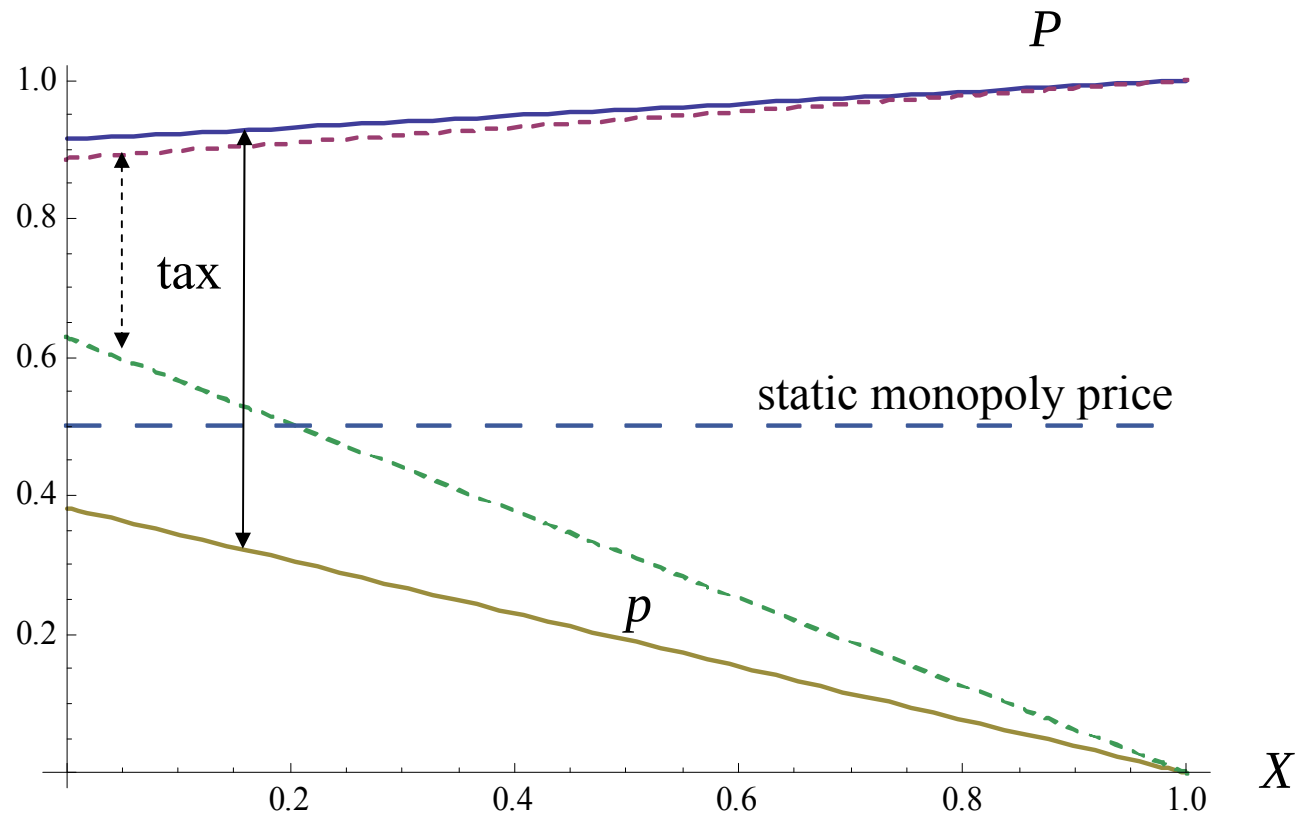


Fig. 2: Comparing different mix of strategies for ($r = 0.05 = c \Rightarrow X_\infty = 1$):

1. **Tax vs price** (dashing) and Nash equilibrium
2. **Taxes vs output quotas** with the government having the first move
(= outcome of the tax-price game too)

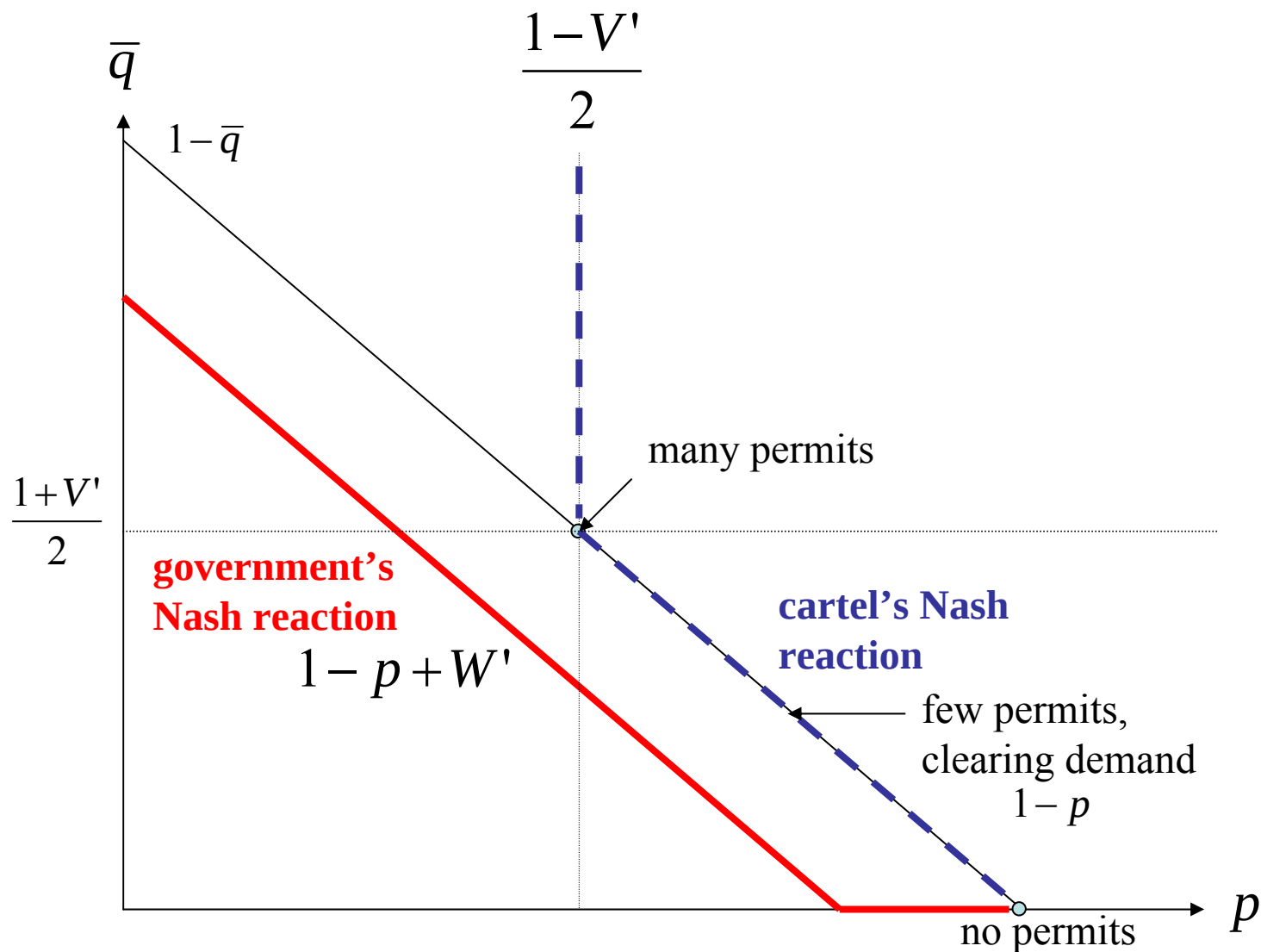


Fig. 3: Nash reaction functions for a permit issuing government and a price setting cartel.

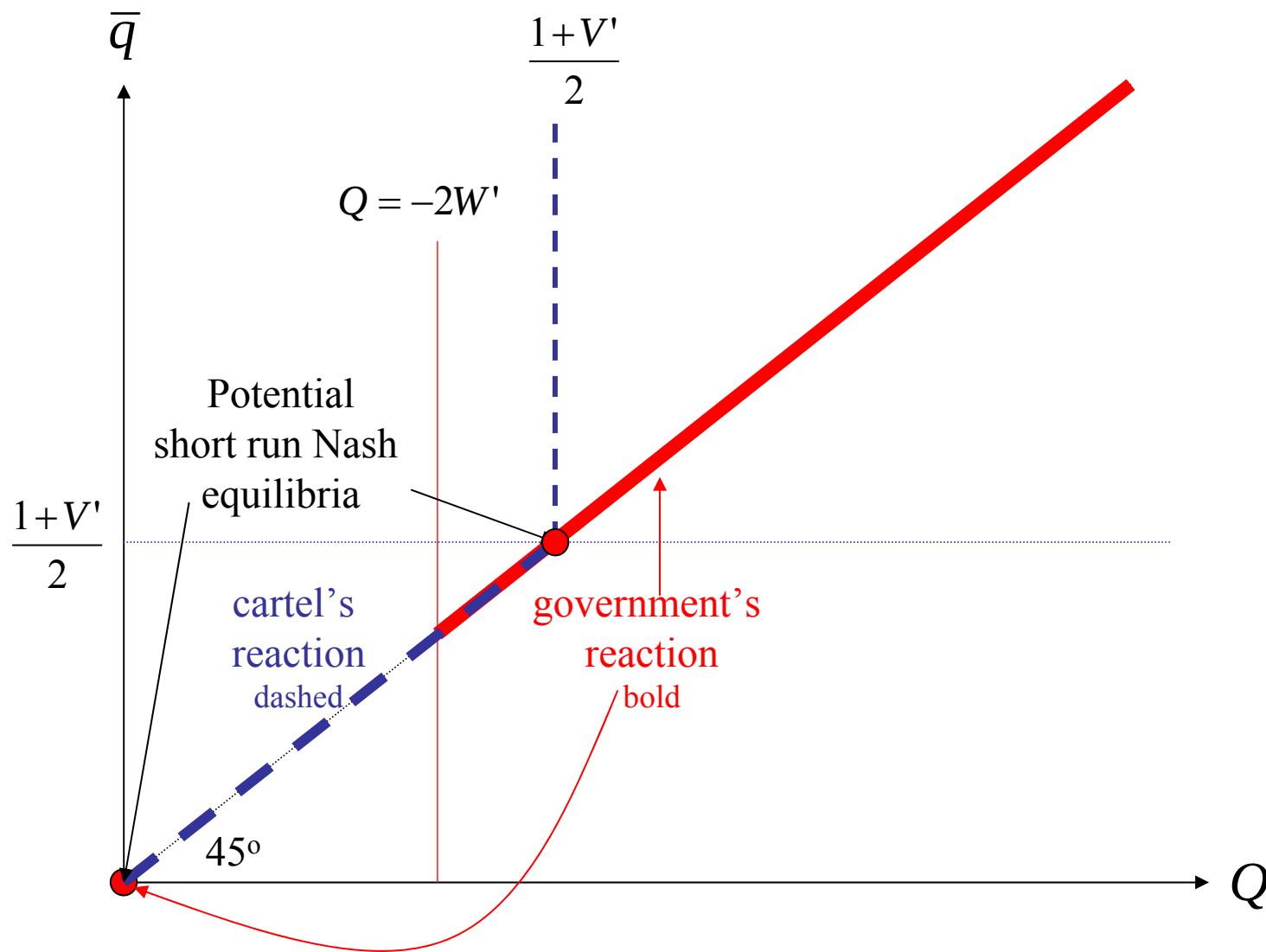


Fig. 4: Nash reactions between a permit issuing government and a quantity setting cartel depending on the shadow prices (V' and W').

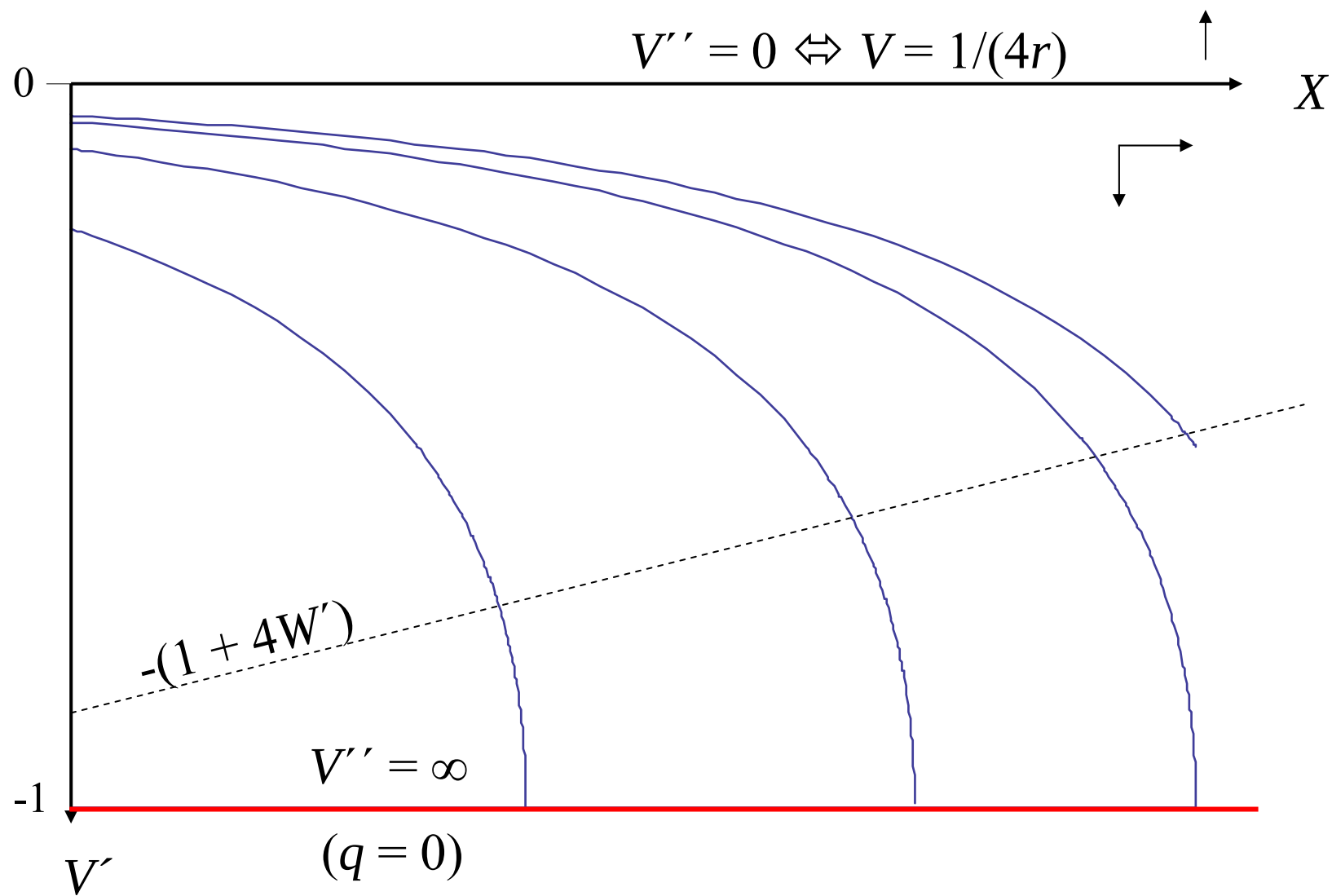


Fig. 5: Phase diagram for price setting cartel facing a permit issuing government, which can commit in the short run.